



*Instructor Comments: In order to aid in the students investigation, this GS applet has been created that simulates the motion of the fly. When “Animate Point” is chosen, the point moves at a constant rate counter-clockwise. While the students work in their groups, the point will be animated and left in motion. At various times in the lesson, students will be probed with questions such as: “Why does your graph describe the covariation of the two quantities? Does your graph depend on the speed of the fan? Why did you choose to draw a ‘smooth’ graph opposed to linear segments? What does the curvature of your graph tell us about how the height of the fly varies with the distance traveled by the fly?” The questioning will be designed to have the students explicitly discuss the covariation of the quantities in terms of directional change and corresponding amounts of change (e.g., for successive equal changes of total distance, the vertical distance above the center of the fan increases and the change of vertical distance decreases over the first fourth of a rotation). The instructor will use the applet throughout this exploration by choosing the “Show Height” button to reveal the measurements of the vertical distance of the fly above the center of the fan. This will be intended to promote a connection between the numerical, contextual, and graphical representations of the covarying quantities.*

*After discussing the directional covariation of the situation, my next phase of instruction will be to introduce the unit circle. First, the students will be asked, "How far is the bug away from the center of the fan in radians?" (Slide 17). The students will be expected to discuss that regardless of the location of the fly, it is always the length of a radius away from the fan. Also, this does not depend on the radius of the fan, although the linear measurement corresponding to the length of a radius may vary. At this time, the "Show Coordinates" button of the applet will be chosen to reveal a coordinate system.*

*The coordinate system on the applet is designed such that if the length of a radius is increased, the coordinate system is increased accordingly to maintain a circle of radius one unit. The students will be asked, "What is the unit of measurement if this is the coordinate system?" The intention will be that the students engage in discussions that lead to their recognizing that the length of a radius can be used as the unit of measurement for vertical and horizontal positions relative to the center of the circle, and that this approach is analogous to the use of a radian to measure an angle. If the fly's distance traveled around the fan is measured in a number of radians and the radius of the fan is increased, the coordinates remain the same for a chosen distance traveled. Hence, the vertical distance of the fly above the center of the fan measured as a fraction of a radius remains the same value. However, the linear measurement of this vertical distance will vary if the radius of the circle varies.*