Resources Video 3 (Box Problem)

The Box Problem is used to introduce many of the big ideas that continue throughout the semester. It requires students to pay particular attention to the quantities involved and asks students to examine and explain the co-variation of these quantities. Student must also model the situation using formulas, graphs and tables and be able to make connections between them. The teacher should use this problem to lay the foundation for developing a problem-solving mentality in their students.

In this problem, students should create an open-topped box by cutting squares out of the corners of a piece of paper. Specifically, they are asked to create the box that will have the greatest possible amount of volume. The purpose of asking the problem in this way is that it gives students a reason to look at the different quantities involved in calculating the volume of the box and how changing one of the quantities affects the others, known as co-variation. The teacher should allow students to struggle with deciding how to create their boxes. Some students may come up with a formula relating the quantities while others go through a method of guessing and checking. Allow students to use whichever method they choose, but be sure to require that students give some sort of justification for why they think their box has the largest volume.

After the boxes have been created, the teacher should lead the class in a discussion about the boxes. The discussion should be about the quantities involved in the situation – what they are, which are relevant, what restrictions they might have, which are changing, and how they are changing together. At this point, it may be useful for the teacher to use the **Explore Box applet**. **(Pull this up on computer screen)** It is impossible to create a continuous view of how the quantities change using the only the pieces of paper. This applet allows students to see that view. The teacher can use this applet to ask students questions about how certain quantities change together. For example, a teacher could ask students how the length of the side of the cutout and the length of the box co-vary. They can then use this applet to change the length of the cutout and allow students to see what happens to the length of the box (do this on the computer screen). Here, they would be able to see that as the length of the side of the cutout increases, the length of the box decreases. The teacher can use this applet to help students understand how these quantities are changing together and encourage students to generalize these relationships into formulas as asked for in the worksheet.

Notice, though, that it is still hard for students to visualize an amount of volume using this applet. As the teacher moves into a discussion about how the length of the side of the cutout and the volume of the box change together, they should use the **Explore Box with Cubes applet. (Pull this up on computer screen)** This applet allows students to visualize how the volume is changing as the length of the cutout increases by using the boxes that you see below (**point using mouse)**. Each of these boxes represents one cubic inch of volume. Notice that as the length of the side of the cutout increases, the volume of the box increases and more of the boxes are being filled in. After a certain point though, as the cutout length continues to increase, the volume of the box decreases back to zero. (**Demonstrate using applet).** This applet allows students to begin to examine the

relationship between the two quantities in a continuous way and can help students begin to build a mental picture of what the graph representing these two relationships would look like.

To help students get a better idea of what the graph will look like, the teacher can then utilize the **Exploring Intervals Applet (Pull this up on computer screen).** This applet allows the teacher to focus students on specific amounts of the quantities and specific changes in them. If the teacher clicks on "Show interval 1", that the first part of the graph appears – when the length of the cutout is between 0 and 0.5 inches. Notice that you can adjust the length of the side of the cutout on the paper, and as you do that, it again traces out the volume as the cutout increases. Using this applet, the teacher can either go step by step through the creation of the graph or they can use the piece of paper to continuously trace out the volumes with associated cutout lengths. (**Many things in this paragraph should be done using the applet. All of this video should be of the computer).**

As a teacher, you should decide how many of the applets to use based on the needs of your class. There are two additional applets available to use as well – one looks into more detail at the graph of the volume with respect to the length of the side of the cutout while the other can be used to help students understand how the formula is developed.

Back to Kacie. This problem concludes by having students use the representations that they've created to find associated quantity values. For example, students are asked to find the volume given a specific cutout length, or the possible cutout lengths given a specific volume. This allows students to gain a further understanding of the representations they've created and how they relate the quantities involved.

Be sure to check out the videos tab for classroom examples of how this problem was implemented. Remember, the goal is for students to focus on quantities, how they co-vary, and for students to begin learning how to act as problem-solvers.